Time Series Probability Model on the Network dynamics for Physical Intelligence

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Abstract

This paper proposes a novel time-series probability model for physical intelligence based on latent network dynamics. In the present model, internal state of the physical intelligence on continuous real time space is represented as a wave on the network dynamics. This formulation solves the problems in existing recurrent relation based models, such as infinite expansion of the recurrence, implicit representation of the state in the learned transition rule. Trajectory of behavior can be a result of surfing on the wave of network dynamics in the proposed model.

Keywords: Network Dynamics, Time-Series, Probability model, Stochastic Process, Physical Intelligence.

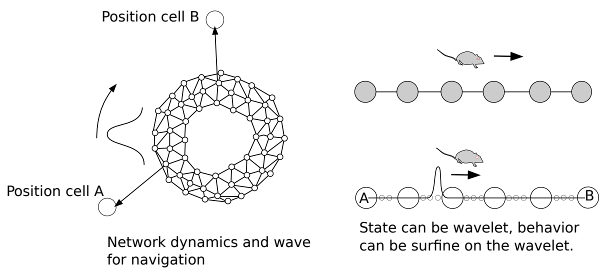

Figure 1: Relation between network dynamics and state transition for behavior in the proposal.

1 Introduction

This paper propose a novel time-series probability model for physical intelligence. In nature physical intelligence must deal with continuous time series to control their body and observe the outer world. To decide the behavior need to be taken, the model need to have internal state that generalize the memory of the past behavior and observation, and those of current. To represent such time-series with internal state, recurrent model such as Recurrent Neural Networks, including Transformers is commonly employed. However, these recurrent model have the following problem: (1) infinite length of the series, (2) implicit definition of the state.

(1) is the problem recurrent model cannot eliminate. In practice, rigorous calculation of the recurrent model needs infinite expansion of the recurrence, which make the computation cost intractable. We have the problem (2) because recurrent model does not represent the state itself, but represent its transition rule in stead. That is, it learns the parameter for transition, and the state is determined as a result of application of the transition. This nature makes it difficult to separate state transition and state definition, that prevents us to expand the model to modular structure as in mammals' brain.

To deal with these problems, this paper focus on network dynamics, which is a generalization of the wave equation. Wave equation describes how the wave travels the space. In modern physics, state representation of the wave and its transition rule is completely separated as an Hamiltonian Mechanics. Therefore, we can define state explicitly and assure the existence of the stable internal state. Having such stable state, we can pass it to another part of the system that makes modular structured expansion possible. Intuitively in the proposal, state corresponds to the wave, and its transition corresponds to movement of the wave. That is, any behavior of physical intelligence can be regarded as a result of surfing on the wave, as described in Fig. 1. Wave is generated by the oscillation of the network as in the left of the picture. In addition, this paper establish the whole dynamics with in probability model, that enables us apply learning algorithms and sampling method to derive the model. Similar idea that employs internal dynamics has been proposed as an Continuous Attractor Network [2]. Novelty of the paper to this existing work is that, separating definitions of state and transition, and giving the way to complete within probability model.

2 Network Dynamics of the Model

This paper employs probability model as foundation of the learning theory. There are various merits to take probability modeling, such as capability of uncertainty, generation of unknown observation (generative capability), tolerance to over fitting, general representing capability in mathematical point of view, and so on. Having those merits is the reason for the paper to employ probability model.

Probability model can be defined using Boltzmann Distribution, which is written as following.

$$p(\mathbf{x}) = Z \exp\left(-H(\mathbf{x})\right) \tag{1}$$

H is called energy that defines how frequent the state $\mathbf{x}$ can occur. Z is a partition function that normalizes the function in the right hand of the equation to make the function p satisfy the probability distribution. Most of practical probability model can be defined by Boltzmann Distribution because for a given distribution q we can take $H(\mathbf{x}) = -\log q(\mathbf{x})$, provided that there is no point of $\mathbf{x}$ on which $q(\mathbf{x}) = 0$ exactly, hardly occurred case in learning model. Therefore, this paper focus on H itself rather defining probability directly.

For H, this paper introduce the following formulation that describes network dynamics (wave equation) for the reason mentioned in the introduction, to represent internal state as the wave.

$$H = \frac{1}{2}\langle p, Lp\rangle + \frac{1}{2}\langle v, v\rangle, \tag{2}$$

where $L \in \mathbb{R}^{D\times D}$ is symmetric graph laplacian, $\langle\cdot,\cdot\rangle$ denotes inner product, $p, v \in \mathbb{R}^D$ are state vectors corresponds to displacement and its velocity. D is indices set for the axis, defines the dimension of the space as $|D|$. State of the $\mathbf{x}$ thus is in the vector space with a dimension $2|D|$, where half of the axes corresponds to p and the other to v. Because L is semi-positive definite, and the second term of the equation clearly non-negative, H is lower bounded, thus, it can compose probability distribution as an energy of the exponent in the equation (1).

To sample from the Boltzmann distribution with H, Langevin dynamics (one of markov chain monte carlo) can be used. It forms following dynamics [1] for the present case of (2).

$$\frac{\mathrm{d}v}{\mathrm{dt}} = -v + \sqrt{2T}\frac{\mathrm{d}W}{\mathrm{dt}}, \frac{\mathrm{d}p}{\mathrm{dt}} = -Lp + \sqrt{2T}\frac{\mathrm{d}W}{\mathrm{dt}}, \tag{3}$$

where W is winner process independent for each equation that introduce noise to the system, and T is a temperature that determines the level of the noise (that corresponds to how random the behavior is.) Noise is needed to explore every possible state, in other words, to reset the current state and start a new behavior.

This dynamics will sample a state from any energy level with the probability defined in (1). However, there is another solution to this sampling problem. Because all state having the same value of energy is sampled with equal probability, therefore, if we draw a line that runs on the surface of certain constant energy level, all of the point is a valid samples from the distribution (1) as well. There is well-know solution to draw such line for the present energy H in (2) called Hamiltonian mechanics defined by the following dynamics.

$$\frac{\mathrm{d}v}{\mathrm{dt}} = -\frac{\partial H}{\partial p}, \frac{\mathrm{d}p}{\mathrm{dt}} = \frac{\partial H}{\partial v}. \tag{4}$$

It becomes following equation for the case of present H.

$$\frac{\mathrm{d}v}{\mathrm{dt}} = -Lp, \frac{\mathrm{d}p}{\mathrm{dt}} = v, \tag{5}$$

More rigorously, we can prove any transition with (5) keep the distribution $p(\mathbf{x})$ invariant, in other word, the transition holds balance condition needed for a Markov Chain Monte Carlo sampling method to sample from the given probability distribution.

Dynamics of (5) and (3) have different effect, (5) samples from the surface of the same energy level that enables the behavior be described as surfing on the wave, while (3) samples from any energy level that enables take a try and error and begin another sequence of behavior (surfing). Thus, (5) must be combined with (3), to have the energy transition in the system. In other words, (3) derives invariant and ergodic sampler, but (5) derives invariant but not ergodic sampler. Therefore, the following dynamics is a possible solution.

$$\begin{aligned}\frac{\mathrm{d}v}{\mathrm{dt}} &= -\lambda Lp - v + \sqrt{\frac{2T}{\lambda}}\frac{\mathrm{d}W}{\mathrm{dt}},\\ \frac{\mathrm{d}p}{\mathrm{dt}} &= \lambda v - Lp + \sqrt{\frac{2T}{\lambda}}\frac{\mathrm{d}W}{\mathrm{dt}}.\end{aligned} \tag{6}$$

where $\lambda > 0$ is a coefficient to compensate the difference of time axis scale between equations (5) and (3). Due to nature of stochastic process, we need a factor $\sqrt{\lambda}^{-1}$ in the third terms of the both equation (see a text book of stochastic process for more in detail.) although it is counter intuitive compared to the case in ordinal deterministic differential equation. (6) can be simulated using the same algorithm, Euler–Maruyama method method as in Langevin Dynamics [1].

(5) is suitable to real world, because most of the physical phenomenon like Newtonian dynamics are written in the same form, hamiltonian dynamics. Thus, just using (3) can sample from any distribution, however, it is supposed that most of the trial by the noise term in (3) (second W related term) will not effective to search on the probability state space of real world. (5) can draw a line along with the narrow probability valley line, as a result, it offers effective sample to search on the state space. To select another path at the branch of such valleys, gradient descent effect of the first term (3) will provide the decision and that of the second term provide trial to unseen choice in the past to beyond small hills in the state space. (5) will, from another point of view, provide continuous movement, like in navigation on the physical space, while (3) provide optimization effect like taking the best pose suitable to given situation, keeping the pose sitting on the chair so as not to falling.

Now time-series predicting problem becomes a sampling problem, not the inference problem give the state. If we prepare output projection function $G_i : \mathbb{R}^{2D} \to Y$, where Y is a space of observable values and $i \in D$ is a index for each unit, it can project the corresponding wave phase to each output. I.e., setting $y_i = G_i(p, v)$, we can configure G_i or learn G_i so that it can fire if preferred wave form appears in the dynamical state consists of p, v. Because wave travel, in time dependent manner, y_i will fire in order of index i. Samples can yield learning method [5] as it can estimate gradient of the loss function, we can learn the parameter without packing off the model to time axis like in the way taken for RNN. See the left of Fig. 1 to obtain intuitive understanding of this manner. Position cell means the cell that fire at appropriate position (in navigation analogy), that shows an example of using the proposal to generate a sequence of place for navigation in time dependent and real time.

3 Projection to the real world and discussion

As described in Fig. 1, the state of the proposed dynamical model can be projected to the outer state like sensor input of the physical body, using ordinal network, like feed forward neural network [3]. Internal state is adjusted by the sensor input, and the internal state will transit to the subsequent state derived by the dynamics. As a result, the model predict preferred state of the input (imagination).

Unlike [2] that define dynamics itself, the state transition is defined as a dynamics to sample the state from probability distribution. Therefore, the proposed formulation gives strong background of the probability theory, having various merits as mentioned in the beginning of this section. For example, free energy principle (FEP) [4] can be applied. Therefore, as in FEP, if a mechanism to control the body so that minimize the difference of imagination and actual observation is given, the proposed dynamical model can be used as a core module of the navigation in real world exploration. One candidate of such mechanism is to introduce a loss function of the predictive coding network [6] into the energy function H.

Sampling based learning rule of the probability model is well described in [5]. The present paper describes only state transition dynamics, however, using the learning method in [5], it is supposed to be possible to apply the present work to the real world problem like robot navigation, or explanation of mammals' brain.

Time scale is limited to the maximum period of the network dynamics. To have long time scale, we must adjust parameters λ, T in (6). Generating long period of the one certain behavior means the intelligence must focus on that behavior. In other words, it cannot take another behavior meanwhile. Such effect taking new behavior is controlled by T (The less T, the less frequency of new behavior like trial and error.) Specifically, if $T = 0$, the dynamics will generate periodic transition without starting another behavior (like just continuing walking regularly.). In that case, λ defines the length of period. Controlling these 2 parameters is another problem that must deal with in the future work.

4 Conclusion

This paper proposed a novel time-series probability model that have the latent network dynamics for physical intelligence. The state and state transition wad separately defined in the formulation, and the model definition was enclosed in the theory of the probability model. The proposed dynamics to run the model described how 2 behavior, trial and error, and sequence of certain behavior can occur, are generalized in one dynamics. Application to the real world problem was also discussed. As future work, simulation of the proposed dynamics will give the detailed property of the proposal, and application of this work to the virtual or real physical environment is excepted to give a new framework of the development of physical intelligence.

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