Proposal of Space-time Probability Model for Intelligence on Physical Space

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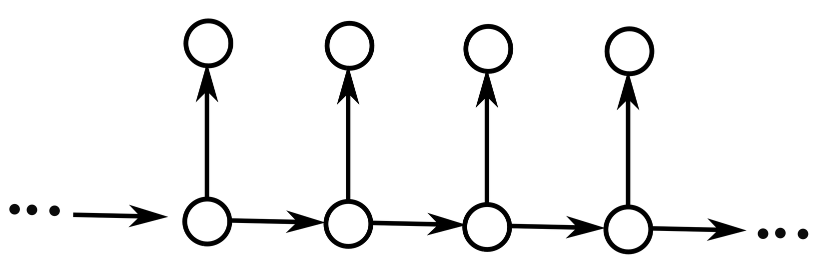
Abstract: this study formalizes a space-time probability model for intelligence in physical space. The novelty lies in the employment of Hamiltonian mechanics, which effectively models the real physical world. While traditional approaches represent time-dependent phenomena as sequences of discrete time points, the present approach models a scalar quantity called Hamiltonian instead, which can describe a wide range of physical phenomena. The system's dynamics are derived from this scalar function, eliminating the need for an explicit sequence of time points that needs general but high-cost computer architecture like Von neumann machine. This approach keeps the model sufficiently simple to be implemented on naive hardware such as field-programmable gate arrays or even biological neurons.

Introduction

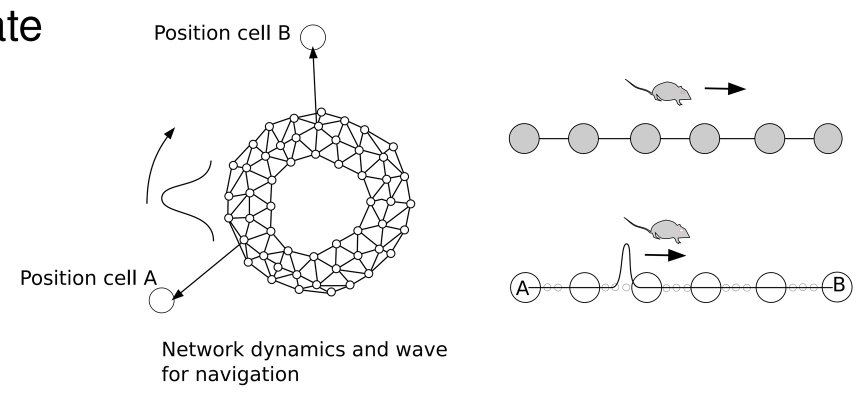
Era of physical bodied Artifitial Intelligence
- LLM (Large Language Model) is core control model [4]

Disadvantage of LLM based model
- Vast amount of energy consumption
- Difficult to immediate reaction response
- Non feedback mechanism for precise manipulate
- Lack of space-time representation



Calculation graph of the recurrent based time-series model
Inifinite expansion of the loop needs copy of cells

Time and space dependency model
- needed for navigation of intteligence
- Reinforcement learning (agent) base
- Time serise model
- Convolution, partial differential equation



Idea of proposal. Wave can travel without time serise graph. And robot move its body along with internal wave

Free Energy Principle [3]
- Under the assumption animals own probability modeling system
- Life is equivalent to decreasing Free energy on the probability

Predictive coding networks [5]
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Research Qeustion.

- How energy minimizing modeling can represent periodic phenonmenon?
- Can periodic phenonmenon be represented without calculation graph?
- How can the model represent time-space dependency?

Approach of the study

- Energy minization => Langevin sampling
- Periodic phenonmenon => Sampling trajectory of hamilton mechanics
- time-space dependencyt => Wave equation with Laplacian

Proposal

Model time and space dependency as scalar function Hamiltonian Mechanism, with 2nd form of Laplacian

$$\phi_P = \frac{1}{2}\langle x_P, Ax_P\rangle + \frac{\alpha_2}{2}\|x_Q\|^2 + \frac{\alpha_3}{2}\|f(x_{Q+P}, x_i)\|^2 + \frac{1}{2}\|x_D\|^2$$

$\langle x_L, y_L\rangle = \frac{1}{|L|}\sum_{i\in L} x_i y_i, \|x_L\|^2 = \langle x_L, x_L\rangle$ First order discretazation of integral

$\Lambda = P + Q + C + D, P* = Q, C* = D$ Set of conjugate cell groups, Conjugate couples, with operator $*$

Probability model with Boltzmann Distribution (Energy based)

$$p(\mathbf{x} = x) = Z\exp(-\phi(x))$$

ϕ : energy, Z : partition function (nomalization)

All internal cognition state of Intelligence is a sample of the probability model (Different from Friston, where neuron represent probability function itself)

=> Langevin dynamics define time evolution

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial\phi}{\partial\mathbf{x}}(x) + \frac{\mathrm{d}W}{\mathrm{d}t}$$

W : wiener process

Time evolution is a result of samping from the probability

Langevin dynamics: make the prob. dist. invariant.

Hamiltonian Mechanics: another transition to make the prob. dist. invariant [2]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \left(\hat{H}\phi\right)(x) \quad \left(\hat{H}\phi\right)_i = \frac{\partial\phi}{\partial\mathbf{x}_{*i}}, \left(\hat{H}\phi\right)_{i*} = -\frac{\partial\phi}{\partial\mathbf{x}_i}, i \in \hat{\Lambda}, \nabla\phi = \frac{\partial\phi}{\partial\mathbf{x}}$$

Linear combination of Hamiltonian Mechanics + Langevin Dynamics: invariant.
<= can be seen in Fokker Planck equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \eta\left(\hat{H}\phi\right)(x) - \xi\partial_\Lambda\phi(x) + \sqrt{\xi}\frac{\mathrm{d}W}{\mathrm{d}t}$$

Constraint on the model likely to show traveling wave: Traveling wave have lower energy with filterfunction

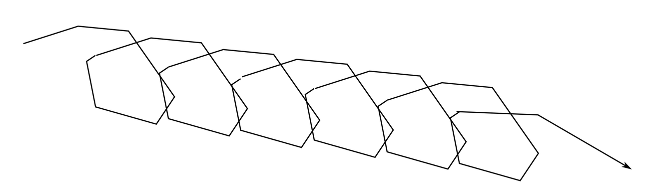
$$f(x) = \left[\sum_{j\in Q+P} S_{Cj}x_j - x_C\right], f: \mathbb{R}^\Lambda \to \mathbb{R}^C, S \in \mathbb{R}^{C\times(Q+P)}$$

$$S_{ij} = \cos(2\pi z_j - 2\pi z_i), j \in P, i \in C$$
$$S_{ij^*} = \sin(2\pi z_j - 2\pi z_i), j \in P, i \in C$$

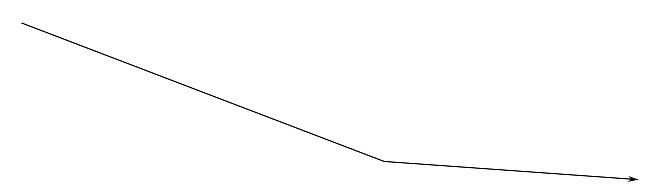
So that wave pattern of traveling wave (d'Alembert's solution)

Hamiltonian Mecanics on laplacian generates wave

Wave most matches to outer hamiltonian mechanics infered as internal state (cognition by predictive coding)



With Hamiltonian Mechanics: Periodic movement is possible. Can represent repeative movement like walking .



Only energy minimization: No preodic movoment occure, because periodic movement conserve energy, not decreasing it.

Biological Plausibility of wave equation in brain <= Oscillating pattern is observed commonly in rat's hippocampus [1]

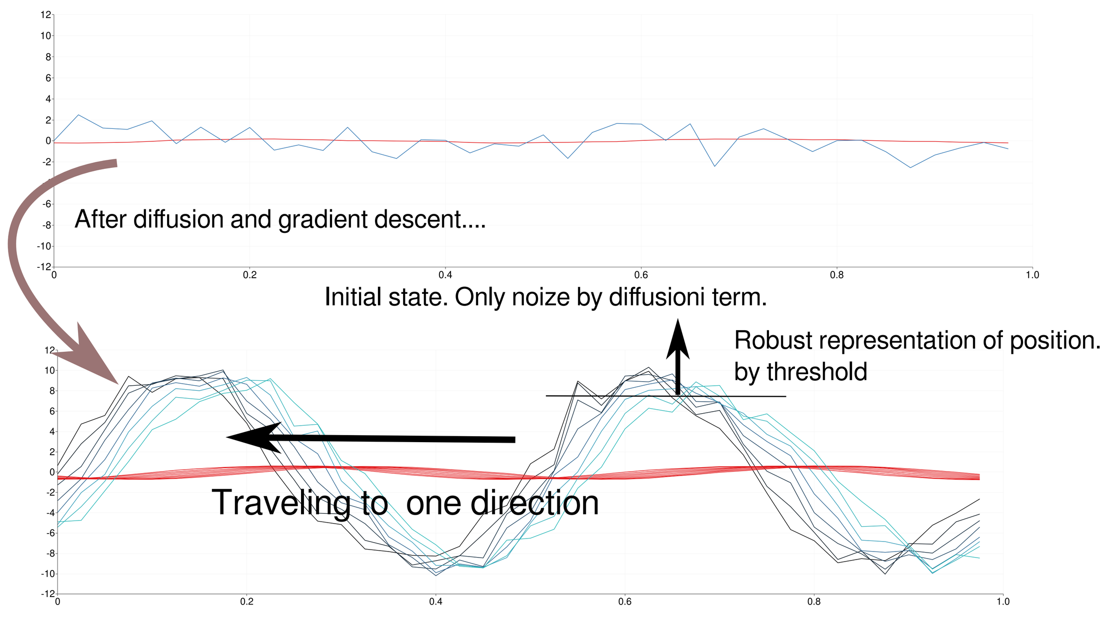
Experiments

Scheme: Euler-Maruyama mthod, 2nd order central finite difference

Nomization: $\max\{c_i | i \in C\} = 1$

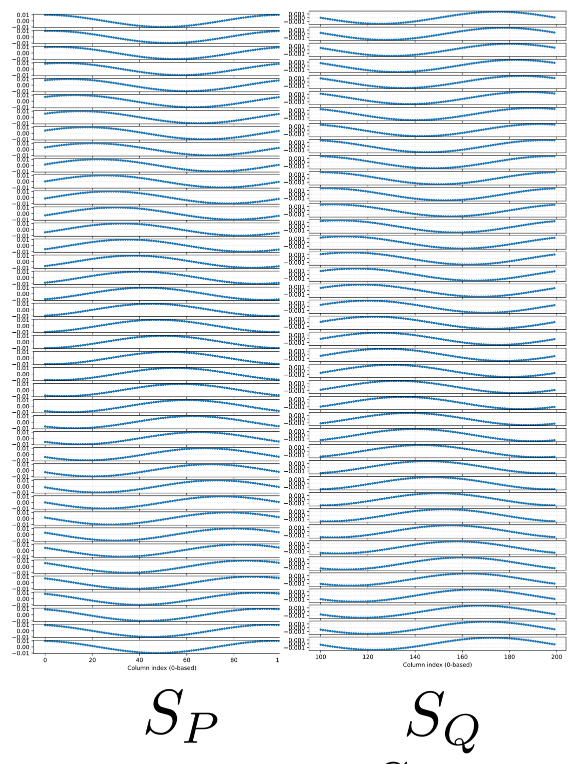
Note that no nomalization to $P, Q, D\ (x_P, x_Q, x_D)$

$\Delta_t = 2^{-18}, \eta = 1, \xi \approx 0.1, \alpha_2 = 1, \alpha_3 = 1000$



State at the undimensional time of 0.34 [-]. Wave corresponding to filter emerge: and start to travel
Emerge: effect of langevin dynamics, Travel: Effect of Hamiltonian Mechanics

Not both direction => time dependency possible



S_P S_Q

Visualization of S

Summary and future

Conclusion:
- Novel Space -time probability model is proposed
- Theory to represent time-dependent phenomenon as a trajectory of sampling => **no need of time serise calculation graph.**
- Hamiltonian Mecahnism explain movement and periodic movemnet on probability model, without time expansion of calculation graph => **can be applied to robot walking model**

[1] Gao+: On Path Integration of Grid Cells Group Representation and Isotropic Scaling, NeurIPS, 2021
[2] Yoshida+, Langevin dynamics neglecting detailed balance condition, 2015, Physical Review
[3] Karl Friston *, James Kilner, Lee Harrison, A free energy principle for the brain, 2006
[4]docs/RT-2: Vision-Language-Action Models Transfer Web Knowledge to Robotic Control, 2023
[5] Predictive coding in the visual cortex: a functional interpretation of some extra-classical receptive-field effects, Nature Neuroscience 1991.